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Determining Effective Thermal Conductivity of Fabrics by Using Fractal Method

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Abstract In this article, a fractal effective thermal conductivity model for woven fabrics with multiple layers is developed. Structural models of yarn and plain woven fabric are derived based on the fractal characteristics of macro-pores (gap or channel) between the yarns and micro-pores inside the yarns. The fractal effective thermal conductivity model can be expressed as a function of the pore structure (fractal dimension) and architectural parameters of the woven fabric. Good agreement is found between the fractal model and the thermal conductivity measurements in the general porosity ranges. It is expected that the model will be helpful in the evaluation of thermal comfort for woven fabric in the whole range of porosity.

Keywords Fabrics · Fractal · Fractal dimension · Thermal conductivity

1 Introduction

The effective thermal conductivity and moisture transfer characteristics of single layer or multilayered fabrics are greatly related to the comfort of clothing [1–3]. As is widely known, heat conduction in porous materials such as fabrics is usually described macroscopically by averaging the microscopic heat transfer processes over a representative elementary volume [4–6]. Therefore, some heat conduction models have been proposed to calculate the effective thermal conductivity for fabrics and other porous media based on the porosity average method [7–10]. However, the disordered and heterogeneous inner structure will result in a great error in the prediction of the effective thermal conductivity as a result of the utilization of the average volumetric porosity.

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The fabric microstructures reveal a considerably disordered distribution of wefts, warps, and channels (or gaps) due to the in-plane displacement when multiple layers of fabrics are stacked together [11]. These pores and their structural distributions are essentially analogous to microstructures in sandstone, to islands or lakes on earth, and to other disciplines [12–14]. Fractal theory is a new subject developed only in recent years, which reveals the unifications between in-order and out-of-order and between determinability and randomicity. Fractal structure has widely appeared in various textile productions, natural or artificial [15–17]. Therefore, it is feasible to apply the fractal theory and technique to the effective thermal conductivity of porous fabrics with N layers.

In this article, we examine plain woven fabrics because they are both widely used and have the representative geometric woven structure. The main purpose of the research is to deal with an effective thermal conductivity model based on the particular fractal characteristics of multilayered plain woven fabrics. Further study will focus on effective thermal conductivity measurements and comparisons between results of model predictions and experimental data.

2 Fractal Geometric Structure of Yarn and Woven Fabric

Previous study indicates that a fractal structure formed by both macro-pores (gaps or channels) between the yarns and micro-pores inside the yarns exists within the woven fabric due to the disordered nature of fabric microstructures. The fractal characteristics of pores in woven fabrics are different from those in other porous materials. Therefore, both these two kinds of pores should be included in a unit cell for fractal analysis. There are two sub-domains within the fabric. One consists of the yarns (warps and wefts) that are woven together to create an interconnected network. A yarn is a bundle of individual fibers held together with thread. The second is a network of empty pores and channels around yarns. For the purpose of simplicity, the unit cell in Fig. 1a is idealized as that shown in Fig. 1d.

The fibers and the squares with the same sectional area are assumed inside the yarn. An equivalent element cell including a single fiber and side air pore is constructed based on the local fractal dimensions along the directions transverse to the heat flux. The fiber section of diameter d_{f} is modeled by a square of the same area. That is, the round fiber section of d_{f} in diameter is converted into the square cross section with an edge length of c_{f}. Moreover, the section of an actual yarn prefers a shape of a course round. To keep the model simple, we assume that the cross section of a yarn is rectangular (Fig. 2b).

The measure of a fractal object $M(L)$ is governed by the scale, L, through a scaling law in the form of [12]

$$M(L) \sim L^{D_{\mathrm{f}}} \tag{1}$$

where "$\sim$" should be read as "scales as." M can be the length of a line, the area of a surface, the volume of a cube, or the mass of an object and D_{f} is the fractal dimension of the object. The macro-pores between yarns and micro-pores inside yarns are

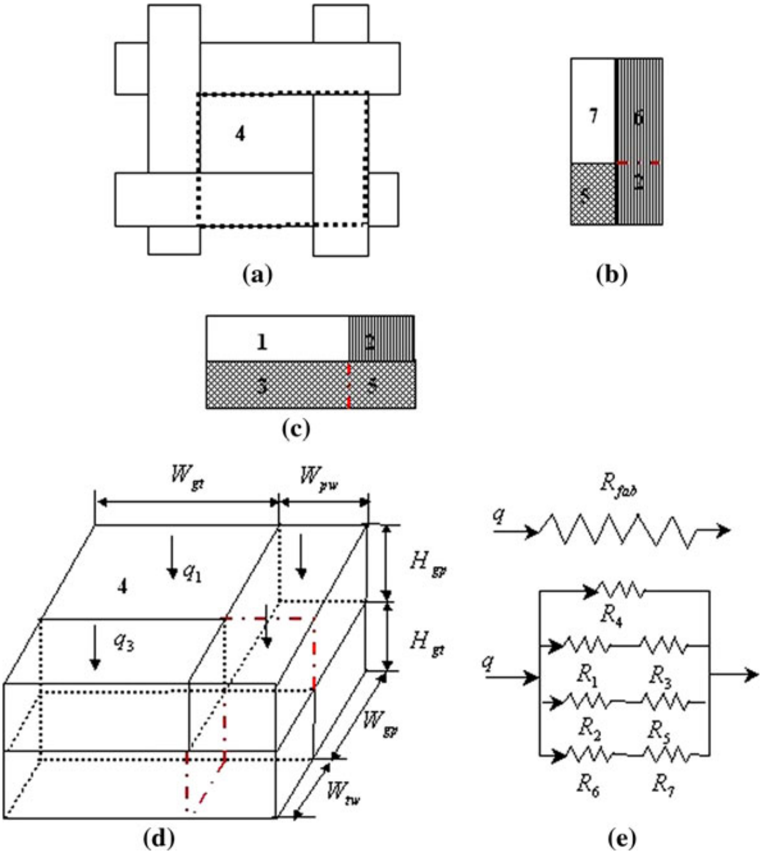

Fig. 1 Structural model of fabric section: (**a**) architecture of the plain woven fabric, (**b**) cross section along the weft direction for the idealized unit cell, (**c**) cross section along the warp direction for the idealized unit cell, (**d**) idealized plain woven unit cell, and (**e**) thermal resistance network

analogous to islands or lakes on earth or spots on engineering surfaces. Thus, the number N of channel pathways should follow the following scaling law [18]:

$$N(L \geq \kappa) = \left(\frac{\kappa_{\max}}{\kappa}\right)^{D_\mathrm{f}} \tag{2}$$

where κ is the pore size and $\kappa_{\max}$ is the maximum size of a pore.

Figure 1 shows the structural model of unit yarn within a plain weave fabric. According to Eq. 1 the pore volume in a unit cell can be expressed as

$$V_{\mathrm{fab}} = L^{D_{\mathrm{fab}}} \tag{3}$$

Thus, the fractal dimension D_{fab} is

$$D_{\mathrm{fab}} = \ln V_{\mathrm{fab}} / \ln L \tag{4}$$

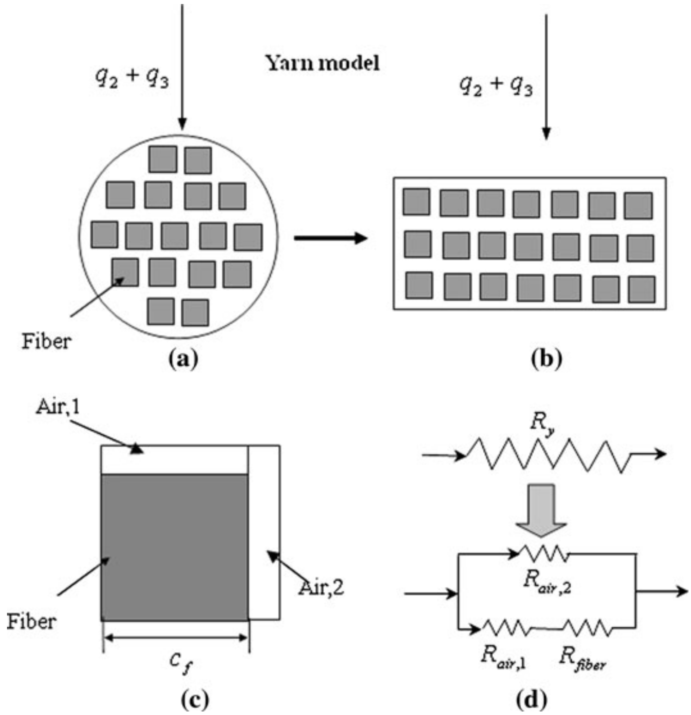

Fig. 2 Structural model of yarn section: (**a**) idealized cross section of a yarn, (**b**) actual cross section of a yarn, (**c**) equivalent cell unit for yarn, and (**d**) heat resistance of unit cell

We know the pore volume fraction is described as

$$\varphi_{\mathrm{fab}} = V_{\mathrm{fab}}/V = L^{D_{\mathrm{fab}}-3} \tag{5}$$

where V is the unit area of cuboids and can be replaced by an equivalent cube volume with a side length of L, in other words, $V = (W_{\mathrm{gt}} + W_{\mathrm{pw}})(W_{\mathrm{tw}} + W_{\mathrm{tw}})(H_{\mathrm{gp}} + H_{\mathrm{gt}}) = L^3$ (Fig. 1d).

3 Determination of Fractal Dimension D_{fab}

The box-counting method [19] in fractal geometry is used to forecast the fractal characters and calculate the local pore fractal dimension D_{fab} of the woven fabric in this study. The box-counting method is based on image analysis of a unit cell (or a sufficiently large cross-section of a sample) along a plane normal to the principal direction of heat flow. In this method, the cross section under analysis is first discretized using cube boxes of size κ. Then the number of boxes $N(\kappa)$ required to completely cover the pore volumes is counted. The pore fractal dimension D_{fab} can be determined by the value of the slope of a linear fit through data on a log–log plot of $N(L \geq \kappa)$

versus κ. The fraction dimension D_{fab} can vary between 1 and 2. The fractal dimension $D_{\text{fab}} = 2$ corresponds to a smooth surface or a plane or a compact cluster.

4 Fractal Model for Effective Thermal Conductivity

It is assumed that any effective thermal property E for a fractal porous media can be expressed as a function of the fractal dimension d_{f} of a section area, each individual thermal parameter E_i of different phases, and the porosity φ of porous media [20,21], i.e.,

$$E = f(E_i, \varphi, d_{\text{f}}) \tag{6}$$

The unit cell derived from woven fabric consists of three parts: channel or gap, weft, and warp. First, the effective thermal conductivity to the transverse direction of the yarn (warp or weft) is calculated based on fractal theory. It is assumed that the weft and warp have the same structural parameters. Fractal theory has been applied to experimental studies of yarn cross section pore structure by Cao et al. [22]. Results show that there are statistical self-similarity characteristics in yarn. A simplified heat conduction model for a real yarn element can be conducted as shown in Fig. 2c and its heat conduction process can be analyzed by the use of a thermal resistance network as shown in Fig. 2d. By use of the method used in an electrical circuit, the effective thermal conductivity of the yarn element is obtained,

$$\frac{1}{R_{\text{y}}} = \frac{1}{R_{\text{air},2}} + \frac{1}{R_{\text{air},1} + R_{\text{fiber}}} \tag{7}$$

$$\lambda_{\text{y}} = \left[1 - \left(1 - \varphi_{\text{y}}\right)^{\frac{1}{2}}\right]\lambda_{\text{g}} + \frac{\lambda_{\text{f}} \cdot \lambda_{\text{g}}}{\lambda_{\text{g}} + \lambda_{\text{f}}\left[\left(1 - \varphi_{\text{y}}\right)^{\frac{1}{2}} - 1\right]} \tag{8}$$

From Eq. 8, it is shown that the effective thermal conductivity of yarn mainly depends on the thermal conductivities of air and fiber and the section void fractions φ_{y}.The effective porosity φ_{y} of yarn can be determined by the effective porosity φ_{fab} of the whole fabric constructed by weft and warp yarns [18].

$$\varphi_{\text{y}} = \frac{\varphi_{\text{fab}} - (V_1 + V_4 + V_7)/V}{(V_2 + V_2 + V_5 + V_6)/V} \tag{9}$$

where V is the volume of a unit cell. $V_1 \ldots V_6$ and V_7 are the volumes of corresponding passages.

Assume that the natural convection and radiation in cell can be neglected because the cell length is usually less than 1 mm and the temperature level in all applications is much lower. The heat flow through the cross section of the unit cell passes through not only the channel but also the weft and the warp. Therefore, heat flow q consists of four sections: (1) air heat conduction q_1 through air channel 4; (2) heat flux q_2 through the series-connection passages between pore passage 1 and weft partial passage 3; (3)

heat flux q_3 through the series-connection passage between pore passage 7 and warp partial passage 6; and (4) heat flux q_4 through warp partial passage 2 and weft partial passage 5.

An analogy to electric circuits was used to analyze the heat transfer properties of the unit cell and the thermal resistance network is shown in Fig. 1e.

$$\frac{1}{R_{\mathrm{fab}}} = \frac{1}{R_4} + \frac{1}{R_1 + R_3} + \frac{1}{R_2 + R_5} + \frac{1}{R_7 + R_6} \tag{10}$$

We obtained the effective thermal conductivity in the direction of the fabric thickness, λ_{fab}, as

$$\lambda_{\mathrm{fab}} = L\lambda_{\mathrm{y}} \left(\frac{\lambda_{\mathrm{g}}}{A\lambda_{\mathrm{y}} + C\lambda_{\mathrm{g}}} + \frac{\lambda_{\mathrm{g}}}{D\lambda_{\mathrm{y}}} + \frac{1}{B+E} + \frac{\lambda_{\mathrm{g}}}{F\lambda_{\mathrm{g}} + G\lambda_{\mathrm{y}}} \right) \tag{11}$$

where $A = \frac{W_{\mathrm{gt}} W_{\mathrm{tw}}}{H_{\mathrm{gt}}}$, $B = \frac{W_{\mathrm{tw}} W_{\mathrm{pw}}}{H_{\mathrm{gp}}}$, $C = \frac{W_{\mathrm{gt}} W_{\mathrm{tw}}}{H_{\mathrm{gp}}}$, $D = \frac{W_{\mathrm{gt}} W_{\mathrm{gp}}}{H_{\mathrm{gt}} + H_{\mathrm{gp}}}$, $E = \frac{W_{\mathrm{tw}} W_{\mathrm{pw}}}{H_{\mathrm{gt}}}$, $F = \frac{W_{\mathrm{pw}} W_{\mathrm{gp}}}{H_{\mathrm{gp}}}$, and $G = \frac{W_{\mathrm{pw}} W_{\mathrm{gp}}}{H_{\mathrm{gt}}}$. Combining Eq. 11 with Eqs. 5, 8, and 9, we find that λ_{fab} is related with the fabric structural size and void fraction dimension D_{fab}.

The algorithm for the present effective thermal conductivity model is summarized as follows:

(1) The fractal dimension D_{fab} is counted by the value of the slope of the best linear fit through the points in this plot.
(2) Evaluate the porosity φ_{fab} of the plain woven fabrics with N layers from Eq. 5.
(3) Measure the architectural parameters, W_{gt}, W_{pw}, W_{gp}, W_{tw}, H_{gp}, and H_{gt} of the plain woven fabrics with N layers, and calculate V, $V_1 \ldots V_7$ from these architectural parameters.
(4) Find λ_{y} from Eqs. 5, 8, and 9.
(5) Finally, obtain the effective thermal conductivity λ_{fab} of multi-layer fabrics from Eq. 11.

5 Validation of the Thermal Conductivity Model

A three-layer woven fabric consisting of Nomex® fiber with different porosity was selected in the experiments. The experimental results were obtained from Textile Thermal Insulation Tester-YG606D produced by Ningbo Textile Instrument Factory. The apparatus was set up according to the GB/T 11048-1989 [23] standard test method. All experiments were performed in an ambient chamber with an air temperature of 20 °C and a relative humidity of 50 %. The samples are cut in a defined square (250 mm × 250 mm) and will be conditioned prior to testing. The linear density of warp and weft is 14.5 tex. The warp and weft density for fabric are 260 per 10 cm and 230 per 10 cm. The other structural parameters such as W_{tw} can be determined with video zoom microscope.

There are many analytical models existing to calculate the effective thermal conductivity of porous material. Nield [24], in applying his model to a porous material,

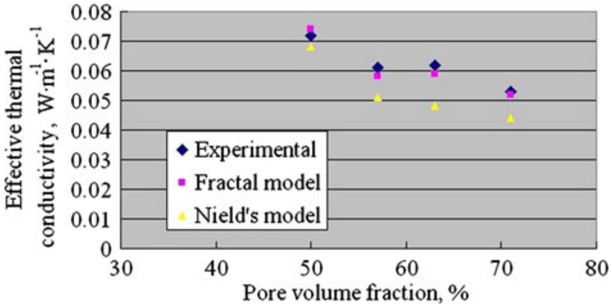

Fig. 3 Relation between the pore volume fraction and the effective thermal conductivity

determined that for specimens where fibers are perpendicular to the heat flow, the thermal conductivity λ can be expressed as

$$\lambda_{\text{fab}} = \lambda_{\text{g}}^{\varphi_{\text{fab}}} \lambda_{\text{fab}}^{1-\varphi_{\text{fab}}} \tag{12}$$

Comparisons between the experimental results and the theoretical calculated values for the effective thermal conductivity in the direction of the thickness of plain woven fabrics are shown in Fig. 3. Generally speaking, the current model predictions are in agreement with experimental values. In comparison, Nield's model underestimates the experiment by 8 % or so. The effective thermal conductivity is very sensitive to the porosity φ_{fab} of the whole fabric. By varying the multilayered fabrics' porosity while keeping other thermal parameters fixed, the effective thermal conductivity of the plain woven fabric increases as the pore volume fraction increases. This is because the effective thermal conductivity of air is smaller than that of the fiber.

6 Conclusion

The fractal model for the effective thermal conductivity is based on the fractal characteristics of pores in multilayered fabrics. In this model, λ_{fab} is a function of the volume fraction dimension and structural parameters of the fabric. The model predictions for the thermal conductivity are compared with experimental data, and good agreement is found between them. Compared with other prediction models, the fractal model avoids errors in the prediction of the effective thermal conductivity due to the average volumetric porosity assumption.

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